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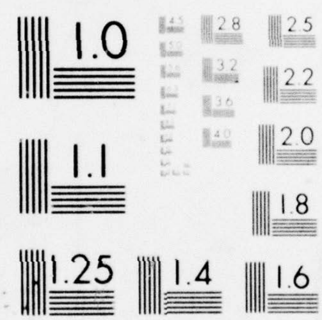

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MRC Technical Summary Report 1673

POSITIVE EIGENFUNCTIONS FOR A CLASS OF SECOND-ORDER ELLIPTIC EQUATIONS WITH STRONG NONLINEARITY

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September 1976

(Received March 1, 1976)

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POSITIVE EIGENFUNCTIONS FOR A CLASS OF SECOND-ORDER ELLIPTIC EQUATIONS WITH STRONG NONLINEARITY †

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ABSTRACT

Let Ω be a bounded domain of $\mathbb{R}^N$. We consider the equation $\mathcal{Q}u(x) + f(x, u(x)) = \lambda u(x)$, $\int_\Omega |u|^2(x)dx = R^2 > 0$, where $\mathcal{Q}$ is a second-order quasilinear elliptic operator whose coefficients have polynomial growth and f essentially satisfies a sign condition. The existence of positive and negative solutions is proved.

AMS(MOS) Subject Classification - 35J20, 35D05, 47H15.

Key Words - Nonlinear eigenvalue problem, Strong nonlinearity, Positive solution.

Work Unit Number 1 - Applied Analysis

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† Supported by the Fonds National Suisse de la Recherche Scientifique.

Sponsored by the United States Army under Contract No. DAAG29-75-C-0024.

POSITIVE EIGENFUNCTIONS FOR A CLASS OF SECOND-ORDER ELLIPTIC EQUATIONS WITH STRONG NONLINEARITY†

Philippe Clément

The aim of this note is to prove the existence of positive and negative solutions of the eigenvalue problem:

(1) $Au = \lambda u$ with the L^2 norm of u, $\|u\|$ being a prescribed constant $R > 0$.

Here A is a second-order elliptic operator defined on a bounded domain Ω of $\mathbb{R}^n$, with a strong nonlinearity in its lowest order term. We assume that A has a variational structure. The corresponding problem

(2) $Au = f$ has been considered by several people [1], [2], [3], [4]. In these papers, only a divergence structure condition for A is assumed, however for the eigenvalue problem, such a hypothesis is too weak in general. Moreover [1], [2], [4] also deal with equations of higher order. In such situations, one can expect the existence of infinitely many distinct pairs of solutions with a prescribed norm, provided that A is odd, but not necessarily the existence of positive or negative solutions. Therefore it is convenient to consider the second-order case in itself.

1. Statement of the results

Let Ω be a bounded domain of $\mathbb{R}^N$.

$a: \Omega \times \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}$ measurable for all $(t, \xi) \in \mathbb{R} \times \mathbb{R}^N$ and $C^1(\mathbb{R} \times \mathbb{R}^N; \mathbb{R})$ for almost all $x \in \Omega$, satisfying the following conditions:

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a1) $a(x, -t, -\xi) = a(x, t, \xi)$ for a. a. x in Ω, all (t, ξ) in $\mathbb{R} \times \mathbb{R}^N$.

a2) $a(x, 0, 0) = 0$ for a. a. x in Ω.

If we denote by $a_i(x, t, \xi)$, $i = 1, \ldots, N$, the partial derivative of a with respect to ξ_i and $a_0(x, t, \xi)$ the partial derivative of a with respect to t, we assume:

a3) there exists $C > 0$ and $2 \leq p < \infty$ such that:

$$|a_\alpha(x, t, \xi)| \leq C(k(x) + |t|^{p-1} + |\xi|^{p-1}), \text{ where } |\xi| = |\sum_{i=1}^{N} \xi_i^2|^{\frac{1}{2}} \text{ for } \alpha = 0, 1, \ldots, N, \text{ and } k \in L^{p/p-1}(\Omega).$$

a4) Leray-Lions conditions:

i) $\sum_{i=1}^{N} [a_i(x, t, \xi) - a_i(x, t, \xi')] (\xi_i - \xi_i') > 0$ for $\xi \neq \xi'$

ii) $\lim_{|\xi| \to +\infty} (\sum_{i=1}^{N} a_i(x, t, \xi)\xi_i)/(|\xi| + |\xi|^{p-1}) \to \infty$

for a. a. x in Ω and $|t|$ bounded.

Let $f: \Omega \times \mathbb{R} \to \mathbb{R}$ be measurable for all $t \in \mathbb{R}$ and continuous for almost all x in Ω, satisfying:

f1) $\operatorname*{ess\,sup}_{x \in \Omega} \sup_{|s| \leq t} |f(x, s)| \leq K(t)$, for all $t \in \mathbb{R}$

f2) $f(x, t)t \geq 0$ for almost all x in Ω.

<u>Remark</u>. If f doesn't depend on x, f1) is trivially satisfied and f2) is purely a sign condition. Observe that by our assumptions on a, $\varphi(u) := \int_\Omega a(x, u, Du)dx$ is $C^1(W_0^{1,p}, \mathbb{R})$.

We consider the following equation:

$$(3) \qquad \int_\Omega \sum_{i=1}^N a_i(x,u,Du)\,D^i v\,dx + \int_\Omega f(x,u)v\,dx = \lambda \int_\Omega uv\,dx$$

$$\text{for all } v \in W_0^{1,p}(\Omega) \cap L^\infty(\Omega) \text{ and } \int_\Omega |u|^2 dx = R^2 > 0 .$$

By a positive (resp. negative) solution of (3), we mean a pair $(\lambda, u) \in \mathbb{R} \times W_0^{1,p}(\Omega)$ such that u is ≥ 0 a.e. (resp ≤ 0 a.e.), $f(u)$ and $f(u)u$ are in $L^1(\Omega)$, and satisfy (3) and (4):

$$(4) \qquad \int_\Omega \sum_{i=1}^N a_i(x,u,Du)\,D^i u\,dx + \int_\Omega f(x,u)u\,dx = \lambda R^2 .$$

Our main result is:

<u>Theorem 1.</u> If a and f satisfy a1 - a4 and f1 - f2, and if $\lim_{\|u\|_{W^{1,p}} \to \infty} \varphi(u) = +\infty$, then for all $R > 0$, (3) possesses at least one positive and one negative solution.

2. <u>Proof</u>.

a) First observe that if $u \in W_0^{1,p}(\Omega)$, then $u^+ = \sup(u, 0)$ belongs to $W_0^{1,p}(\Omega)$ as well as $|u| = \sup(u, -u)$. Let $\Omega^+ \subset \Omega$ be the support (as a distribution) of u^+. We have $\varphi(|u|) = \varphi(u)$. Indeed $\varphi(|u|) = \int_\Omega a(x, |u|, D|u|)dx = \int_{\Omega^+} a(x,u,Du)dx + \int_{\Omega-\Omega^+} a(x,-u,-Du)dx = \int_{\Omega^+} a(x,u,Du)dx + \int_{\Omega-\Omega^+} a(x,u,Du)dx = \varphi(u)$, by a1).

b) Since we are looking for positive solutions, without loss of generality we can assume that f is odd. Indeed we can replace f by $\tilde{f}$ defined by $\tilde{f}(x,t) = f(x,t)$ for $t \geq 0$ and $\tilde{f}(x,t) = -f(x,-t)$ for $t < 0$. The negative case is similar.

By a1), if f is odd, and (λ, u) is a positive solution, then $(\lambda, -u)$ is a negative one, so we can restrict ourselves to the case of positive solutions, with f odd.

c) We shall first prove the theorem under the additional assumption that f is bounded, let $\psi(u) := \int_\Omega dx \int_0^{u(x)} f(x,t)dt$. It is known that $\psi \in C^1(W_0^{1,p}, \mathbb{R})$. Clearly $\psi \geq 0$. By the compact imbedding of $W_0^{1,p}(\Omega)$ in $L^2(\Omega)$ $(p \geq 2)$, if $u_n \rightharpoonup u$ in $W^{1,p}$, then $u_n \to u$ in L^2 and therefore $\psi(u_n) \to \psi(u)$. We know that $\varphi \in C^1(W_0^{1,p}, \mathbb{R})$. Moreover $\varphi'(u)$, the Frechet derivative of φ at u is bounded by a3) and satisfies [see 5, p. 183]: $u_n \rightharpoonup u$ in $W_0^{1,p}$, $\varphi'(u_n) \rightharpoonup v$ in $(W_0^{1,p})'$ and $\overline{\lim}\ \langle \varphi'(u_n), u_n - u \rangle \leq 0$ imply $v = \varphi'(u)$ and $\lim \langle \varphi'(u_n), u_n \rangle = \langle \nu, u \rangle$ †. It easily follows that $\varphi' : W_0^{1,p} \to (W_0^{1,p})'$ is of type (P)‡, [6] and therefore $\varphi : W_0^{1,p} \to \mathbb{R}$ is sequentially weakly lower semicontinuous. Hence $\varphi + \psi$ is also s.w.l.s. For $R > 0$, let $\alpha := \inf_{\nu \in S^R} \varphi(\nu) + \psi(\nu)$, where $S^R := \{u \in W_0^{1,p} \mid \int_\Omega |u|^2 dx = R^2\}$.

Clearly $S^R \neq \emptyset$, so $\alpha < \infty$. Let $u_n \in S^R$ such that $\varphi(u_n) + \psi(u_n) \downarrow \alpha$. Since $\psi \geq 0$ and by the assumption of the theorem, $\|u_n\|_{W^{1,p}} \leq C$, for some $c > 0$.

We have $\varphi(|u_n|) + \psi(|u_n|) = \varphi(u_n) + \psi(u_n)$ and $\| |u_n| \|_{W^{1,p}} = \|u_n\|_{W^{1,p}} \leq C$. By the compact imbedding of $W^{1,p}$ into L^2, and the reflexivity of $W^{1,p}$ there exists $u \in W_0^{1,p}$ with $\int_\Omega |u|^2 dx = R^2$ and $u \geq 0$ such that $|u_n| \rightharpoonup u$ in $W^{1,p}$ and $|u_n| \to u$ in L^2. Therefore, by our previous remark, $\varphi(u) + \psi(u) \leq \underline{\lim}\, \varphi(|u_n|) + \psi(|u_n|) = \alpha$. But $u \in S^R$, so $\varphi(u) + \psi(u) = \alpha$. Hence the minimum of

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† $\langle\ ,\ \rangle$ shall denote the duality between $W_0^{1,p}$ and $(W_0^{1,p})'$.

‡ $u_n \rightharpoonup u$ in $W_0^{1,p}$ implies $\overline{\lim}\ \langle \varphi'(u_n), u_n - u \rangle \geq 0$.

$\varphi + \psi$ on S^R, is achieved at u. Moreover $\varphi + \psi \in C^1(W_0^{1,p}, \mathbb{R})$, $u \to \frac{1}{2} \int_\Omega |u|^2 dx$ is $C^1(W_0^{1,p}, \mathbb{R})$ and the Frechet derivative of the latter function is $\neq 0$ since $u \neq 0$. Thus, by the well-known "Lyusternik principle", there exists $\lambda \in \mathbb{R}$, such that (λ, u) is a positive solution of (3).

d) For $n \in \mathbb{N}$, define $f_n(x,t) = f(x,t)$ if $|f(x,t)| \leq n$, $f_n(x,t) = n$ if $f(x,t) > n$ and $f_n(x,t) = -n$ if $f(x,t) < -n$. Let $\psi_n(u) := \int_\Omega dx \int_0^{u(x)} f_n(x,t)dt$. Since f_n are bounded, we know that for each $n \in \mathbb{N}$, there exists $(\lambda_n, u_n) \in \mathbb{R} \times W_0^{1,p}$, a positive solution of (3), where f is replaced by f_n. We claim that $\|u_n\|_{W_0^{1,p}} \leq C$ for some $C > 0$. Indeed as in c), we have: $\varphi(u_n) + \psi_n(u_n) = \inf_{v \in S^R} \varphi(v) + \psi_n(v)$. Let $u_0 \in S^R \cap L^\infty(\Omega)$. Then $\varphi(u_n) + \psi_n(u_n) \leq \varphi(u_0) + \psi_n(u_0) \leq \varphi(u_0) + \int_\Omega dx \int_0^{u_0(x)} f(x,t)dt < \infty$. By the coercivity of φ and the positivity of ψ_n we get $\|u_n\|_{W^{1,p}} \leq C$. By the reflexivity of $W_0^{1,p}$ and the compact imbedding of $W_0^{1,p}$ into L^2 we can extract a subsequence, still denoted by u_n, such that $u_n \rightharpoonup u$ in $W^{1,p}$ and $u_n \to u$ in L^2. So $u \in S^R$ and u is positive a.e.

We will prove that λ_n is bounded. Indeed, assume that there is a subsequence $\lambda_n \downarrow -\infty$, then we get:

$$\frac{1}{\lambda_n} \langle \varphi'(u_n), u_n \rangle + \frac{1}{\lambda_n} \int_\Omega f_n(x, u_n) u_n \, dx = R^2 . \tag{5}$$

But $\varphi'(u_n)$ is bounded, since u_n is, so $\frac{1}{\lambda_n} \langle \varphi'(u_n), u_n \rangle \to 0$. But then $\frac{1}{\lambda_n} \int_\Omega f_n(x, u_n) u_n \, dx \leq 0$ and $R^2 > 0$ imply that for n big enough we get a contradiction. Next assume now that $\lambda_n \uparrow \infty$, for a subsequence. We have, for all $v \in W_0^{1,p}$:

(6) $$\frac{1}{\lambda_n}\langle \varphi'(u_n), v\rangle + \frac{1}{\lambda_n}\int_\Omega f_n(x, u_n) v\,dx = \int_\Omega u_n v\,dx \; .$$

In particular for $v = u_n$ we get:

(7) $$\int_\Omega \frac{1}{\lambda_n} f_n(x, u_n) u_n \,dx \leq C \quad \text{for some } C > 0 \; .$$

By assumption f1) and f2), for each $\delta > 0$, there exists $K_\delta > 0$ such that $|f(x,t)| \leq K_\delta + \delta f(x,t)t$. [2]. Hence $|f_n(x,t)| \leq K_\delta + \delta f(x,t)t$. By using (7), this shows that the sequence $\frac{1}{\lambda_n} f_n(x, u_n)$ is equiintegrable and since $\frac{1}{\lambda_n} f_n(x, u_n) \to 0$ a.e. in Ω, by Vitali's theorem, $\frac{1}{\lambda_n} f_n(x, u_n) \to 0$ in L^1 and by (6), for $v \in W_0^{1,p}(\Omega) \cap L^\infty(\Omega)$, we get: $\int_\Omega uv\,dx = 0$. Hence by density, $\int_\Omega |u|^2 dx = 0$, a contradiction. Thus λ_n is bounded and we can extract a subsequence $\lambda_n \to \lambda \in \mathbb{R}$.

Since u_n is bounded in $W_0^{1,p}$ and $\varphi' : W_0^{1,p} \to (W_0^{1,p})'$ is bounded, $\varphi'(u_n)$ is bounded and we can extract a subsequence converging weakly to $w \in (W_0^{1,p})'$: $\varphi'(u_n) \rightharpoonup w$ in $(W_0^{1,p})'$. We have $\int_\Omega f_n(x, u_n) u_n \,dx = \lambda_n R^2 - \langle \varphi'(u_n), u_n\rangle \leq C$. By using the same argument as before, there exists a subsequence $f_n(x, u_n) \to f(x,u)$ in L^1 . Moreover, since $f_n(x, u_n) u_n \geq 0$, by Fatou's lemma, $f(x,u)u \in L^1$. Therefore for all $v \in W_0^{1,p} \cap L^\infty$, we have:

(8) $$\langle w, v\rangle + \int_\Omega f(x,u) \;\; dx = \lambda \int_\Omega uv\,dx \; .$$

Now, we define $v_n := \inf(u, n)$. It is known that $v_n \in W_0^{1,p} \cap L^\infty$, $v_n \to u$ in $W_0^{1,p}$ and by Lebesgue's dominated convergence theorem and the fact that $f(x,u)u \in L^1$, $f(x,u)v_n \to f(x,u)u$ in L^1 . By putting $v = v_n$ and passing to the limit we have:

(9) $$\langle w, u\rangle + \int_\Omega f(x,u)u\,dx = \lambda R^2 .$$

Now, we are done provided that we prove that $w = \varphi'(u)$. But $u_n \to u$ in $W_0^{1,p}$, $\varphi'(u_n) \to w$ in $(W_0^{1,p})'$, so it is sufficient to prove that $\overline{\lim}\ \langle \varphi'(u_n), u_n - u\rangle \le 0$. We have $\overline{\lim}\ \langle \varphi'(u_n), u_n - u\rangle = \overline{\lim}\ \langle \varphi'(u_n), u_n\rangle - \langle w, u\rangle = \overline{\lim}\ [\lambda_n R^2 - \int_\Omega f_n(x,u_n)u_n\,dx] - [\lambda R^2 - \int_\Omega f(x,u)u\,dx] = \int_\Omega f(x,u)u\,dx - \underline{\lim} \int_\Omega f(x,u)u\,dx - \underline{\lim} \int_\Omega f_n(x,u_n)u_n\,dx \le 0$, by Fatou's lemma. This completes the proof of the theorem. □

<u>Remark 1.</u> With the same arguments, we can allow a more general right hand side and consider the equation:

(10) $$\int_\Omega \sum_{i=1}^N a_i(x,u,Du)D^i v\,dx + \int_\Omega f(x,u)v\,dx = \lambda \int_\Omega g(x,u)\,v\,dx$$

for all $v \in W_0^{1,p} \cap L^\infty$ with the condition $\int_\Omega |u|^2 dx = R^2 > 0$ replaced by the condition $\int_\Omega dx \int_0^{u(x)} g(x,t)dt = R > 0$. In this case if $g\colon \Omega \times \mathbb{R} \to \mathbb{R}$ satisfies the Caratheodory condition, and

g1) $g(x,t)t > 0$ for a. a. X in Ω

g2) there exists $1 < q < \infty$ and $C > 0$ such that $|g(x,t)| \le C(1 + |t|^{q-1})$ and if we assume that p in the hypothesis a3) satisfies $1 < p < \infty$, we can state the following generalization of Theorem 1:

<u>Theorem 1'</u>. If a, f, g satisfy $a_1 - a_4$, $f_1 - f_2$, $g_1 - g_2$ and if we have

i) the injection of $W^{1,p}$ into L^q is compact

ii) $$\lim_{\|u\|_{W^{1,p}}} \int_\Omega a(x,u,Du)dx = +\infty$$

iii) $\lim\limits_{\|u\|_{L^q}} \int_\Omega dx \int_0^{u(x)} g(x,t)dt = +\infty$.

Then the equation (10) possess at least one positive and one negative solution $(\lambda, u) \in W_0^{1,p}(\Omega)$ with $f(x,u)$ and $f(x,u)u \in L^1$, for all $R > 0$.

Remark 2. In the simple case $-\Delta u + f(u) = \lambda u$, $u \in W_0^{1,2}(\Omega)$, our condition on f reduces to: i) $f(0) = 0$ ii) $\varliminf\limits_{t\to+\infty} \frac{f(t)}{t} \geq -c$ for some $c > 0$. If we assume further that $f(u) = \tilde{f}(u)u$, $\tilde{f}(t) \geq 0$, $\tilde{f} \in C^1(\mathbb{R})$, it follows from [7], that there exists an unbounded connected set C^+ of positive solutions in $\mathbb{R} \times C^{1,\alpha}(\Omega)$ $(0 < \alpha < 1)$, containing $(\lambda_0, 0)$ in its closure, where λ_0 is the first eigenvalue of $-\Delta h + \tilde{f}(0)h = \lambda h$, $h|_{\partial\Omega} = 0$. If we suppose that $\lim\limits_{t\to\infty} \tilde{f}(t) = +\infty$, it follows easily that the projection of C^+ on $\mathbb{R}$ contains the interval $[\lambda_0, \infty)$. Therefore in this case the above equation possesses positive solutions with arbitrary norm in $L^2(\Omega)$ and for all $\lambda \geq \lambda_0$. It can happen that all positive solutions are in C^+; it is true, for example, if $\tilde{f}'(t) \geq 0$ for all $t \in \mathbb{R}$. For the case $\tilde{f}'(t) > 0$, see [8]; the case $\tilde{f}'(t) \geq 0$ can be handled by using a simiiar argument to [9]. In this particular situation C^+ is a C^1 curve of solutions in $C^{2,\alpha}(\Omega)$, parametrized by λ and unbounded in $L^2(\Omega)$. Concerning the regularity of the solutions, let us mention that if $f \in C^0(\mathbb{R})$, $f(0) = 0$ and f is monotone, then $u \in C^{1,\alpha}(\Omega)$. For a different approach of this case, see [10].

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UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: 1673
2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER: (9) Technical
4. TITLE (and Subtitle): (6) POSITIVE EIGENFUNCTIONS FOR A CLASS OF SECOND-ORDER ELLIPTIC EQUATIONS WITH STRONG NONLINEARITY.
5. TYPE OF REPORT & PERIOD COVERED: Summary Report, ~~no specific~~ reporting period
6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): (10) Philippe Clement
8. CONTRACT OR GRANT NUMBER(s): (15) DAAG29-75-C-0024
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Mathematics Research Center, University of Wisconsin, 610 Walnut Street, Madison, Wisconsin 53706
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS: U. S. Army Research Office, P.O. Box 12211, Research Triangle Park, North Carolina 27709
12. REPORT DATE: (11) Sep[redacted]1976
13. NUMBER OF PAGES: 10 (12) 13p.
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): (14) MRC-TSR-1673
15. SECURITY CLASS. (of this report): UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Nonlinear eigenvalue problem, Strong nonlinearity, Positive solution.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Let Ω be a bounded domain of $\mathbb{R}^N$. We consider the equation $\mathcal{A}u(x) + f(x, u(x)) = \lambda u(x)$, $\int_\Omega |u|^2(x)dx = R^2 > 0$, where $\mathcal{A}$ is a second-order quasi-linear elliptic operator whose coefficients have polynomial growth and f essentially satisfies a sign condition. The existence of positive and negative solutions is proved.

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